The value of normal derivations in the realm of explanations

Francesca Poggiolesi[1]

IHPST, UMR 8590 CNRS, Université Paris 1 Panthéon-Sorbonne

Abstract. The concept of explanation is and has long been the object of deep and wide philosophical debates; in particular it is the notion of causal explanation that has for decades dominated the general attention, e.g. see [11]. Beside the debate on causal explanation, in recent years another type of explanation has gained attention, namely *mathematical explanation*. The expression *mathematical explanations* is an umbrella term that indicates several different phenomena; in this context, we use it to refer to those mathematical proofs that not only show the theorem they prove to be true, but that they also reveal the *reasons* why the theorem it true. The idea that certain mathematical proofs have an explanatory power has been shown to be widespread amongst mathematicians (e.g. see [4]) and to have a long and illustrious philosophical pedigree (e.g. see [3] and [9]). Moreover it is a type of mathematical explanations that has been having a central role in the recent literature on the subject. *To date there has been a tendency to approach the topic of mathematical explanations by investigating the distinction between explanatory and non-explanatory proofs. This is very natural since it is widely acknowledge that some proofs are explanatory whilst other are not.* [1, p. 3]

In the attempt of better understanding mathematical explanatory proofs, some scholars have drawn an analogy with normalized derivations in natural deduction calculi, e.g. see [2,8]. This analogy rests on a feature that both mathematical explanatory proofs and normalized derivations share, namely a complexity's increase from the assumptions to the conclusion of proofs/derivations. On the one hand, one of the main features of explanatory proofs amounts to the fact that they explain the theorem they prove by providing grounds or reasons that are simpler than the theorem they prove. On the other hand, normalized proofs typically satisfy the subformula property[1] and the subformula property can be seen as the formalization of this idea of complexity's increase from the premisses to the conclusion (e.g. see [10]). Although, for several reasons,[2] normalized derivations cannot be considered as a proper formalization of explanatory mathematical proofs, they nevertheless represent a first step towards this direction.

In this talk, the main aim is to deepen the analysis on the relationships between mathematical explanatory proofs and normal derivations; we will do that by proposing a novel model for mathematical explanations according to which when a mathematical proof is (thought of as) explanatory, then there exists a way to formalize it with a normal derivation where the undischarged assumptions are less complex than the conclusion. This modeling of explanatory proofs will involve the use of theorems or mathematical definitions (that occur in the mathematical proof

[1] At least under certain conditions.

[2] E.g. see [6]

in key positions) as rules of the derivation (e.g. see [5]), as well as the extension of the notion of logical complexity to the level of concepts (e.g. see [7]).

References

1. S. Baron, M. Colyvan, and D. Ripley. A counterfactual approach to explanation in mathematics. *Philosophia Mathematica*, 28:1–34, 2020.
2. E. Casari. Matematica e verità. *Rivista di Filosofia*, 78(3):329–350, 1987.
3. P. Kitcher. Explanatory unification. *Philosophy of Science*, 48(4):507–531, 1981. URL: http://www.jstor.org/stable/186834, doi:10.2307/186834.
4. P. Mancosu. Mathematical explanation: Problems and prospects. *Topoi*, 20(1):97–117, January 2001. URL: http://link.springer.com/article/10.1023/A3A1010621314372, doi:10.1023/A:1010621314372.
5. S. Marin, D. Miller, E. Pimentel, and M. Volpe. Synthetic inference rules for geometric theories. *Submitted*, 1:1–28, 2021.
6. F. Poggiolesi. On defining the notion of complete and immediate grounding. *Synthese*, 193:3147–3167, 2016.
7. F. Poggiolesi. Explanations in mathematics: an analysis via formal proofs and conceptual complexity. *Submitted*, pages 1–34, 2022.
8. A. Rumberg. Bolzano's theory of grounding against the background of normal proofs. *Review of Symbolic Logic*, 6(3):424–459, 2013.
9. Mark Steiner. Mathematical explanation. *Philosophical Studies*, 34(2):135–151, August 1978. URL: http://link.springer.com/article/10.1007/BF00354494, doi:10.1007/BF00354494.
10. A. S. Troelstra and H. Schwichtenberg. *Basic Proof Theory*. Cambridge University Press, 1996.
11. J Woodward. Scientific explanation. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. 2017.