On the proof theoretic strength of circular reasoning

Anupam Das

University of Birmingham
a.das@bham.ac.uk

Cyclic and *non-wellfounded* proofs are now a common technique for demonstrating metalogical properties of systems incorporating (co)induction, including modal logics, predicate logics, type systems and algebras. Unlike usual proofs, non-wellfounded proofs may have infinite branches: they are generated *coinductively* from a set of inference rules. Naturally, such 'proofs' may admit fallacious reasoning, and so one typically employs some global correctness condition inspired by ω-automaton theory.

A key motivation in cyclic proof theory is the so-called 'Brotherston-Simpson conjecture': are cyclic proofs and inductive proofs equally powerful? Naturally, the answer depends on how one interprets 'equally powerful', e.g. as provability, proof complexity, logical complexity etc., as well as on the logic at hand. In any case it is interesting to note that the tools employed in cyclic proof theory are often bespoke to the underlying logic, yielding a now myriad of techniques at the interface between several branches of mathematical and computational logic.

In this talk I will discuss a line of work that attempts to understand the expressivity of cyclic proofs via forms of proof theoretic strength. Namely, I address predicate logic in the guise of first-order arithmetic, and type systems in the guise of higher-order primitive recursion, and establish a recurring theme: circular reasoning buys precisely one level of 'abstraction' over inductive reasoning. Along the way we shall see some of the aforementioned interplays in action, in particular exploiting techniques from proof theory, reverse mathematics, automaton theory, metamathematics, rewriting theory and higher-order computability.